MATH 8 – Sample Final

This test is in two parts. On part one, you may not use a calculator; on part two, a calculator is necessary. When you complete part one, you turn it in and get part two. Once you have turned in part one, you may not go back to it.

PART ONE - NO CALCULATORS ALLOWED

(1) Find each of the following:
(Note: answers to inverse trig. problems should be in radians, not degrees)

(a) $\sin^{-1}(-1) =$ $-\frac{\pi}{2}$

(b) $\tan^{-1}(0) =$ 0

(c) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) =$ $-\frac{\pi}{6}$

(d) $\sin^{-1}\left(\frac{1}{2}\right) =$ $\frac{\pi}{6}$

(e) tan 330° $-\frac{1}{\sqrt{3}}$

(f) $\cos^{-1}\left(\frac{-\sqrt{2}}{2}\right) =$ $\frac{3\pi}{4}$

(g) $\sec\left(\frac{5\pi}{6}\right) =$ $-\frac{2}{\sqrt{3}}$

(h) $\csc(\pi) =$ undefined

i) $\cos^{-1}\left(\cos\left(\frac{3\pi}{2}\right)\right) =$ $\frac{\pi}{2}$

(j) $\tan\left(\tan^{-1}(1/3)\right) =$ $\frac{1}{3}$

(2) Fill in the blank to complete the identity.

(a) $\sin 2\theta =$ $2\sin\theta\cos\theta$

(b) $\cos^2 x =$ $1-\sin^2 x$ or $\frac{1+\cos 2x}{2}$

(c) $\sin(\theta/2) =$ $\pm\sqrt{\frac{1-\cos\theta}{2}}$

(d) $\cos(\alpha+\beta) =$ $\cos\alpha\cos\beta-\sin\alpha\sin\beta$

MATH 8 – Sample Final Exam – Part Two

Fill in the blanks. In problems 1 - 7 fill in the blank with the most appropriate answer

(1) $\sin(-\theta) =$ $-\sin\theta$

(2) The graph of the polar curve $r=4\sin\theta$ is a Circle

(3) $\begin{vmatrix} -5 & 3 \\ 2 & -7 \end{vmatrix} =$ 29

(4) The period of $f(x) = \tan(3\pi x)$ is $\frac{1}{3}$

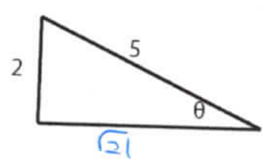
(5) The range of $f(x) = \cos^{-1}x$ is $[0, \pi]$

(6) To graph $f(x) = 2\sin(4x - \pi)$ we would shift the graph of $g(x) = 2\sin(4x)$ $\frac{\pi}{4}$ (how far) to the right

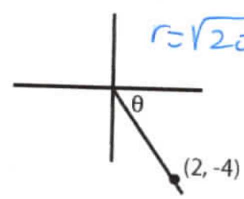
(7) The range of $f(x) = \tan x$ is $(-\infty, \infty)$

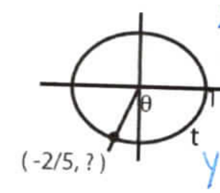
(8) Convert the polar point $(7, 11\pi/6)$ to rectangular coordinates $\left(\frac{7\sqrt{3}}{2}, -\frac{7}{2}\right)$

(9) Given the following figures, find:



$r = \sqrt{20} = 2\sqrt{5}$





$x^2 + y^2 = 1$

$\frac{4}{25} + y^2 = 1$

$y = -\frac{\sqrt{21}}{5}$

(a) $\cos\theta =$ $\sqrt{21}/5$

(b) $\theta \approx$ 23.6 degrees

(c) $\sin\theta =$ $\frac{-4}{2\sqrt{5}} = \frac{-2}{\sqrt{5}}$

(d) $\theta \approx$ -63.4 degrees

(e) $\sin t =$ $\frac{-\sqrt{21}}{5}$

(f) $\theta \approx$ -113.6 degrees

(10) Given the point $(-4, -4)$ in rectangular coordinates, find two different polar representations; one with $r > 0$, the other with $r < 0$.

$r^2 = 32$ $r = \pm 4\sqrt{2}$

$\tan\theta = \frac{-4}{-4} = 1$ Q3

$(4\sqrt{2}, 225°)$

$(-4\sqrt{2}, 45°)$

(11) Given the following matrices:

$A = \begin{bmatrix} 2 & -1 \\ 3 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 1 & 5 \\ 0 & 4 & 3 \\ 1 & -2 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 1 & -3 \\ 3 & 7 \end{bmatrix}$ Find the following, if possible. (If not possible, say so.)

(a) A^{-1}

$\begin{bmatrix} \frac{5}{7} & \frac{-1}{7} \\ \frac{3}{7} & \frac{-2}{7} \end{bmatrix}$

(b) AC

$\begin{bmatrix} -1 & -13 \\ -12 & -44 \end{bmatrix}$

(e) det(B)

$3(18) + 1(-17) = 37$

(12) SOLVE the following equations: $0 \le x < 2\pi$

(a) $\sin 2x = 3\sin x$

$2\sin x\cos x - 3\sin x = 0$

$\sin x(2\cos x - 3) = 0$

$\sin x = 0$ or $\cos x = 3/2$ no soln.

$x = 0, \pi$

(b) $\cos^2(3x) - 1 = 0$

$\cos^2(3x) = 1$

$\cos(3x) = \pm 1$

$3x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

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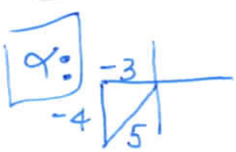
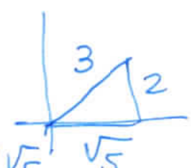
(13)Given $\csc\alpha = -5/4$, $\pi < \alpha < \frac{3\pi}{2}$, and $\beta = \sin^{-1}(2/3)$,

$\sin\beta = 2/3$





$\frac{\pi}{2} < \frac{\alpha}{2} < \frac{3\pi}{4}$

$\frac{\alpha}{2}$ in Q2

Find:

a) $\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 - -3/5}{2}} = \sqrt{\frac{8}{10}} = \frac{2}{\sqrt{5}}$

b) $\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2 \cdot \frac{2}{\sqrt{5}}}{1 - \frac{4}{5}} = \frac{4/\sqrt{5}}{1/5} = \frac{20}{\sqrt{5}} = 4\sqrt{5}$

c) $\cos(\alpha + \beta)$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta = -\frac{3}{5}\frac{\sqrt{5}}{3} - -\frac{4}{5}\frac{2}{3} = \frac{-3\sqrt{5}+8}{15}$

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(14) Verify the identity : $\frac{1-\sin\theta}{\cos\theta} + \frac{\cos\theta}{1-\sin\theta} = 2\sec\theta$

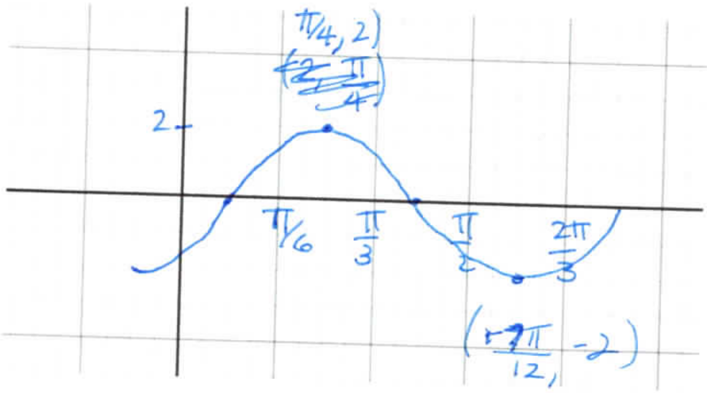
$$\frac{1-\sin\theta}{\cos\theta} + \frac{\cos\theta}{1-\sin\theta} = \frac{(1-\sin\theta)^2 + \cos^2\theta}{\cos\theta(1-\sin\theta)}$$

$$= \frac{1 - 2\sin\theta + \overbrace{\sin^2\theta + \cos^2\theta}^{1}}{\cos\theta(1-\sin\theta)}$$

$$= \frac{2-2\sin\theta}{\cos\theta(1-\sin\theta)} = \frac{2(1-\sin\theta)}{\cos\theta(1-\sin\theta)} = \frac{2}{\cos\theta} = 2\sec\theta$$

(15) Sketch the following graph. (clearly show scale, graph at least one period, label coordinates of highs and lows)

$f(x) = 2\sin\left(3x - \frac{\pi}{4}\right) = 2\sin\left(3\left(x - \frac{\pi}{12}\right)\right)$



period $= \frac{2\pi}{3}$

$\frac{1}{4}$ period $= \frac{1}{4} \cdot \frac{2\pi}{3} = \frac{\pi}{6}$

shift right $\pi/12$

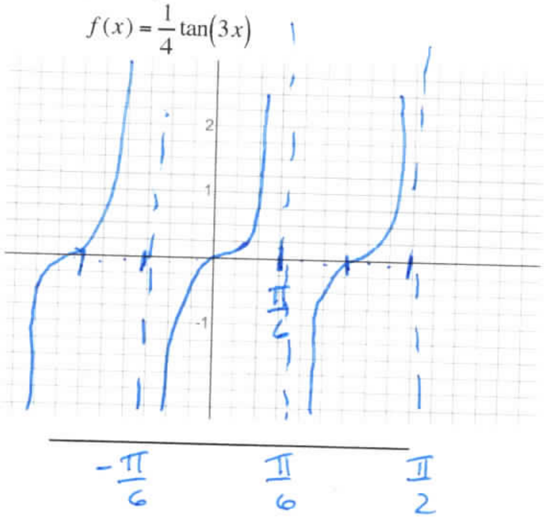
(16) Use Gaussian Elimination OR Cramer's Rule to solve: (no credit if requested method is not used)

$$\begin{cases} 3x - y - z = 8 \\ x + y - 2z = 5 \\ 2x - y + z = 1 \end{cases}$$

$(1, -2, -3)$

(17)
Sketch the following graph. (clearly show scale, graph at least TWO periods, show location of any asymptotes, label 2 points on graph) (5 points)

$f(x) = \frac{1}{4}\tan(3x)$



period $= \frac{\pi}{\omega} = \frac{\pi}{3}$

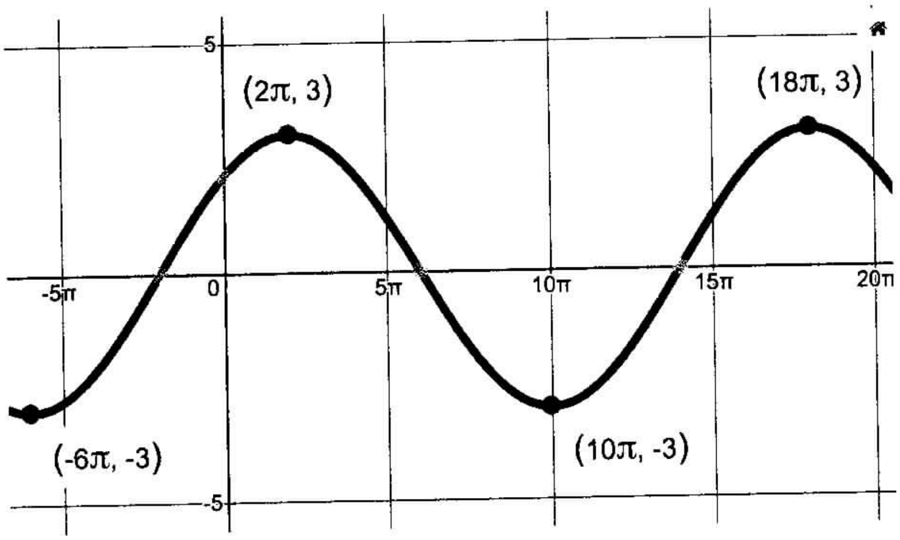
Asymptotes:

$\cos 3x = 0$

$3x = \frac{\pi}{2} + \pi k$

$x = \frac{\pi}{6} + \frac{\pi}{3} k$

(18) Find an equation corresponding the graph below. Check a point.



period = 16π

$\frac{2\pi}{\omega} = 16\pi$

$\omega = \frac{1}{8}$

$f(x) = 3\sin\left(\frac{1}{8}(x+2\pi)\right)$

$f(x) = 3\cos\left(\frac{1}{8}(x-2\pi)\right)$

many other possibilities

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(19) Given the vectors **W** = <-4,-3> and **V** = < 2, 5>, find the following:

a) || **W** || 5

b) **w** • **v** -23

c) Find the direction angle of **w** (exactly) $\tan^{-1}\frac{3}{4} + 180^\circ$

$\tan\theta = \frac{-3}{-4}$ Q3

d) The direction angle of **v** (exactly) $\tan^{-1}\frac{5}{2}$

e) Find b so that <b,7> is orthogonal to **W** $b = \frac{-21}{4}$

$\langle b,7\rangle \cdot \vec{w} = 0 \quad -4b-21=0$

f) Find the angle between **w** and **v** $\cos^{-1}\left(\frac{-23}{5\sqrt{29}}\right)$

$\cos\theta = \frac{\vec{w}\cdot\vec{v}}{\|\vec{w}\|\|\vec{v}\|} = \frac{-23}{5\sqrt{29}}$

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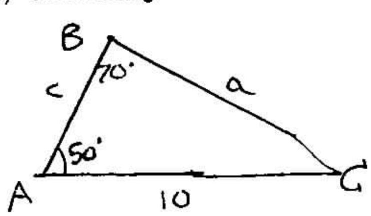
(20) Given triangle ABC with A=50°, B=70° and b=10 inches, find the remaining parts.



$\frac{a}{\sin 50^\circ} = \frac{10}{\sin 70^\circ} \Rightarrow a = \frac{10\sin 50^\circ}{\sin 70^\circ} \approx 8.15$

$C = 180 - A - B \rightarrow C(\text{or } \gamma) = 60^\circ$

$\frac{c}{\sin 60} = \frac{10}{\sin 70^\circ} \Rightarrow c = \frac{10\sin 60^\circ}{\sin 70^\circ} = \frac{5\sqrt{3}}{\sin 70^\circ} \approx 9.22$

Find all solutions to the following equations.

(21) $3\tan^2 x - \sec^2 x - 5 = 0$

$3\tan^2 x - (\tan^2 x + 1) - 5 = 0$

$2\tan^2 x - 6 = 0$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$x = \frac{\pi}{3} + \pi k, \ \frac{2\pi}{3} + \pi k \quad k, \text{integer}$

(22) $\cos(2x) = 2 + 5\cos x$

$2\cos^2 x - 1 = 2 + 5\cos x$

$2\cos^2 x - 5\cos x - 3 = 0$

$(2\cos x + 1)(\cos x - 3) = 0$

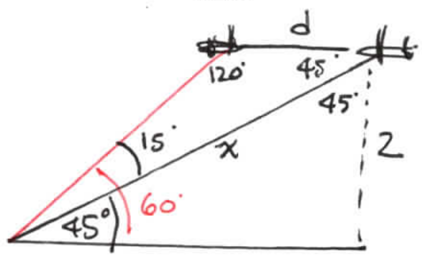
$\cos x = -1/2, \quad \cos x = 3$ No soln

$x = \frac{2\pi}{3} + 2\pi k$

$\frac{4\pi}{3} + 2\pi k$

k integer

(23) A man looks up and sees an airplane flying in his direction at a level altitude of 2 miles. He watches the airplane for a few minutes. During that period of time he notices that the angle of elevation to the airplane changes from 45° to 60°. How far has the plane traveled in that time?



Many approaches possible.

Use big right $\triangle$ to find x

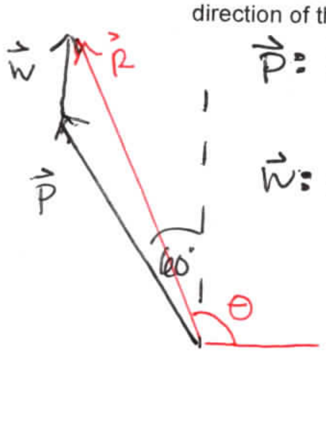
$\frac{2}{x} = \sin 45^\circ \Rightarrow x = \frac{2}{\sin 45^\circ} = 2\sqrt{2}$

Use oblique $\triangle$

$\frac{x}{\sin 120^\circ} = \frac{d}{\sin 15^\circ}$

$d = \frac{x \sin 15^\circ}{\sin 120^\circ} = \frac{2\sqrt{2}\sin 15^\circ}{\frac{\sqrt{3}}{2}} = \frac{4\sqrt{2}\sin 15^\circ}{\sqrt{3}} \approx .845 \text{ miles}$

(24) An airplane is traveling at a constant airspeed of 450 mph in the direction N60°W. If wind is blowing directly northward at a rate of 50 mph, what is the actual speed and direction of the airplane relative to the ground?



$\vec{P}$: $\|\vec{P}\| = 450$, $\theta_P = 150^\circ$

$\vec{W}$: $\|\vec{W}\| = 50$, $\theta_W = 90^\circ$

$\vec{P} = \langle 450\cos 150^\circ, 450\sin 450^\circ \rangle = \langle -225, 225\sqrt{3} \rangle$

$\vec{W} = \langle 0, 50 \rangle$

$\vec{R} = \vec{P} + \vec{W} = \langle -225, 225\sqrt{3} + 50 \rangle$

Actual Speed $= \|\vec{R}\| = \sqrt{(-225)^2 + (225\sqrt{3}+50)^2} \approx 494 \text{ mph}$

$\tan\theta = \frac{225\sqrt{3}+50}{-225}$, Q3

$\theta = \tan^{-1}\left(\frac{225\sqrt{3}+50}{-225}\right) + 180^\circ \approx 117^\circ$

$\approx N(27^\circ)W$